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Math 131 - Fall 2024 - Common Final Exam

Print name: Answers

Section number:______ **Instructor's name:** John G. Del Greco

Directions:

- This exam has 12 questions. Please check that your exam is complete, but otherwise keep this page closed until the start of the exam is called.
- Fill in your name, and your instructor's name.
- It will be graded out of 100 points.
- Show your work. Answers (even correct ones) without the corresponding work will receive no credit.
- You may use a calculator which cannot connect to the internet. The use of any notes or electronic devices other than a calculator is prohibited.
- A formula sheet has been provided with this exam. You may not refer to any other notes during the exam.

Good luck!

Question:	1	2	3	4	5	6	7
Points:	8	8	8	2	10	8	11
Score:							

Question:	8	9	10	11	12		Total
Points:	11	8	8	9	9		100
Score:							

1. (8 points) Find the equation of the tangent line to the graph of $y = 6\sqrt{x} + 8$ at $x = 1$.

$$y' = 6\frac{1}{2\sqrt{x}} = \frac{3}{\sqrt{x}} \quad +2$$

$$\therefore\ y'(1) = \frac{3}{\sqrt{1}} = 3 \quad +2$$

$$y(1) = 6\sqrt{1} + 8 = 14 \quad +2$$

$$y - 14 = 3(x-1)$$

$$\Rightarrow \boxed{y = 3x + 11}$$

+2

2. (8 points) Find the exact value of $p'(1)$ where $p(x) = x^4\, e^{2x}$

$$p'(x) = x^4 (e^{2x})' + (x^4)' e^{2x} \quad +3$$

$$= 2x^4 e^{2x} + 4x^3 e^{2x} \quad +3$$

$$\therefore\ p'(1) = 2(1)^4 e^2 + 4(1)^3 e^2$$

$$= 2e^2 + 4e^2$$

$$= 6e^2 \quad +2$$

$$\boxed{p'(1) = 6e^2}$$

3. A pharmaceutical company is analyzing the concentration of a certain drug in a patient's bloodstream over time. The concentration $C(t)$, in milligrams per liter, is modeled by the function

$$C(t) = \frac{200t}{t^2 + 25}$$

where t represents the time in hours after the drug is administered.

(a) (6 points) Find a formula for the rate of change of drug concentration in the bloodstream at any time t.

$$C'(t) = \frac{(t^2+25)(200) - (200t)(2t)}{(t^2+25)^2}$$ +4

$$= \frac{200t^2 + 5000 - 400t^2}{(t^2+25)^2}$$

$$= \frac{5000 - 200t^2}{(t^2+25)^2}$$

$$\therefore \boxed{C'(t) = \frac{5000 - 200t^2}{(t^2+25)^2}}$$ +2

(b) (2 points) Find the rate of change of drug concentration in the bloodstream after 6 hours have passed. Include units and round your answer to three decimal places.

$$C'(6) = \frac{5000 - (200)(36)}{(36+25)^2}$$

$$\simeq -0.591 \text{ mg/L}$$

$$\therefore \boxed{C'(6) \simeq -0.591 \text{ mg/L}}$$

+1 +1

4. (2 points) Which of the following limits represents the definition of the derivative of the function $f(x) = x^2 + x$? Circle the correct answer. No explanation required.

- $\lim_{h\to\infty} \frac{(x+h)^2 + x - x^2 - x}{h}$
- $\lim_{h\to 0} \frac{(x+h)^2 + (x+h) - (x^2+x)}{h}$
- $\lim_{h\to 0} \frac{(x+h)^2 + x - x^2 - x}{h}$
- $\lim_{h\to 0} \frac{(x+h)^2 - (x)^2}{h}$
- $\lim_{h\to\infty} \frac{(x+h)^2 + (x+h) - (x^2+x)}{h}$
- $\lim_{h\to\infty} \frac{(x+h)^2 - x - (x^2+(x+h))}{h}$

+2

5. (3 points) Suppose that $f(x)$ is a function such that $\int_3^{11} f(x)\,dx = 24$.

(a) (3 points) If $F(x)$ is an antiderivative of $f(x)$ and $F(11) = -15$, find the value of $F(3)$.

$$24 = \int_3^{11} f(x)\,dx = F(11) - F(3)$$ +1

$$\Rightarrow 24 = -15 - F(3)$$ +1

$$\Rightarrow F(3) = -15 - 24 = -39$$

$$\therefore \boxed{F(3) = -39}$$ +1

(b) (4 points) Find the value of $\int_3^{11} (2f(x) + 3)\,dx$

$$\int_3^{11} (2f(x)+3)\,dx = 2\int_3^{11} f(x)\,dx + 3\int_3^{11} dx$$ +2

$$= 2(24) + 3(11-3)$$

$$= 48 + 24$$

$$= 72$$ +2

$$\therefore \boxed{\int_3^{11} (2f(x)+3)\,dx = 72}$$

6. The local water authority is monitoring water usage in a reservoir over several days to ensure an adequate supply. The following table provides the volume w of water, in millions of liters, in the reservoir on different days.

t	3	6	9	12	15
$w(t)$	50.1	48.3	44.8	43.2	40.1

(a) (2 points) Compute the average rate of change of w on the interval $3 \leq t \leq 15$. Show all work and provide appropriate units.

$$\frac{40.1 - 50.1}{15-3} = -0.833 \text{ (millions) L/day}$$

$$= \boxed{-833,333.33 \text{ L/day}}$$

+1 +1

(b) (4 points) Estimate $w'(12)$. Show all work and provide appropriate units.

(right) $w'(12) \cong \frac{40.1-43.2}{3} = -1.0333 = \boxed{-1,033,333.33 \text{ L/day}}$

+2 +1 +1

(left) $w'(12) \cong \frac{44.8-43.2}{-3} = -0.5333 = \boxed{-533333.33 \text{ L/day}}$

(average) $w'(12) \cong \frac{-1.0333 - 0.5333}{2} = -.7833 = \boxed{-783,300 \text{ L/day}}$

(c) (2 points) Use your answer from part (b) to estimate $w(13)$. Show all work and provide appropriate units.

+1 +1

(right) $w(13) \cong w'(12) + w(12) = -1.0333 + 43.2$
$= 42.1667 = \boxed{42,166,700 \text{ L/day}}$

(left) $w(13) \cong w'(12) + w(12) = -0.5333 + 43.2$
$= 42.6667 = \boxed{42,666,700 \text{ L/day}}$

(average) $w(13) \cong w'(12) + w(12) = -0.7833 + 43.2$
$= 42.4167 = \boxed{42,416,700 \text{ L/day}}$

7. Consider the function

$$f(x) = 2x^3 - 12x^2 + 18x + 5.$$

(a) (3 points) Find all critical points of $f(x)$.

$$f'(x) = 6x^2 - 24x + 18 = 6(x^2 - 4x + 3) = \underbrace{6(x-3)(x-1)}_{+1}$$

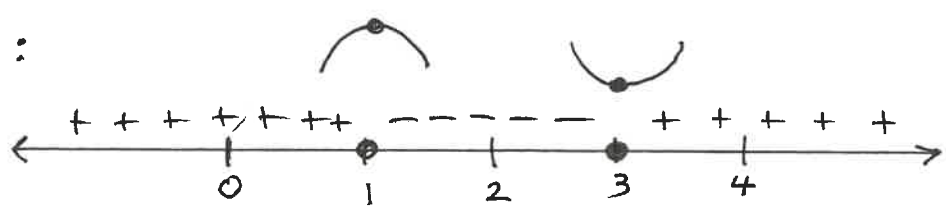
$$\Rightarrow \boxed{\begin{matrix} x = 3 \\ x = 1 \end{matrix}}$$

+2

(b) (3 points) For each of the critical points found in part (a), determine if it is a local maximum, local minimum or neither. You may use either the first or second derivative test to justify your answer.

First Der. Test:

$f'(x)$:



+1

OR

$$\boxed{\begin{matrix} x = 1 \text{ is a local maximum} \\ x = 3 \text{ is a local minimum} \end{matrix}}$$

+2

Second Der. Test:

$f''(x) = 12x - 24$ +1

$f''(1) = 12(1) - 24 < 0 \Rightarrow$ $\boxed{x = 1 \text{ is a local maximum}}$

$f''(3) = 12(3) - 24 > 0 \Rightarrow$ $\boxed{x = 3 \text{ is a local minimum}}$

+2

Parts (c) and (d) below are a continuation of problem 7, and they refer to the same function

$$f(x) = 2x^3 - 12x^2 + 18x + 5$$

used in parts (a) and (b).

(c) (2 points) Find the absolute maximum and absolute minimum of $f(x)$ on the interval $0 \le x \le 5$.

$f(0) = \underline{\underline{5}}$

$f(1) = 2 - 12 + 18 + 5 = \underline{\underline{13}}$

$f(3) = 2(27) - 12(9) + 54 + 5$

$= 54 - 108 + 54 + 5$

$= \underline{\underline{5}}$

$f(5) = 2(125) - 12(25) + 18(5) + 5$

$= 250 - 300 + 90 + 5$

$= -50 + 95 = \underline{\underline{45}}$

$\therefore$ $x = 0$, $x = 3$ are global minima

$x = 5$ is the global maxima

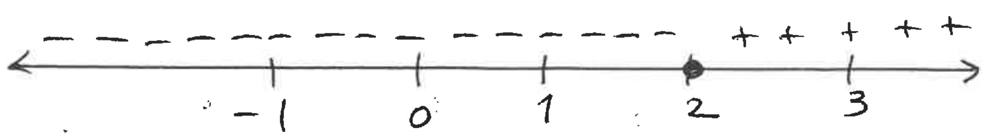
+1

+1

(d) (3 points) Find all value(s) of x for which $f''(x) = 0$ and determine whether or not these value(s) are inflection points or not.

$f''(x) = \underbrace{12x - 24}_{+1} \Rightarrow x = 2$

$f''(x)$:



+1

$\therefore$ $\boxed{x = 2 \text{ is an inflection point}}$

+1

8. The biodiversity $B = f(A)$ of a forest ecosystem is modeled by the function

$$B(A) = 200\ln(5A+2).$$

where B represents the number of species in the ecosystem and A is the area in square kilometers of the ecosystem.

(a) (2 points) Find the rate of change $B'(A)$ of the biodiversity with respect to area.

$$B'(A) = \frac{200}{5A+2}(5)$$

$$= \boxed{\frac{1000}{5A+2}}$$

+2

(b) (6 points) Evaluate $B'(50)$ and interpret your result in the context of the ecosystem. Include appropriate units

$$B'(50) = \frac{1000}{5(50)+2}$$

$$= \frac{1000}{252}$$

$$= 3.97$$

$$\cong 4$$

+3

$\therefore$ If the area is $50\,km^2$, every additional km^2 results in about 4 new species in the eco system.

+3

(c) (3 points) Estimate the area A of the ecosystem if it is known that its biodiversity increases at the rate of 2 species per additional square kilometer. Round your answer to the nearest square kilometer.

$$\frac{1000}{5A+2} \cong 2$$

+2

$$\Rightarrow 1000 \cong 10A + 4$$

$$\Rightarrow 996 \cong 10A$$

$$\Rightarrow A \cong 99.6\ km^2$$

$$\Rightarrow \boxed{A \cong 100\ km^2}$$

+1

OR

$$B(A+1) - B(A) \cong 2$$

+2

$$\Rightarrow 200\ln(5A+7) - 200\ln(5A+2) \cong 2$$

$$\Rightarrow \ln\left(\frac{5A+7}{5A+2}\right) \cong .01$$

$$\Rightarrow \frac{5A+7}{5A+2} \cong e^{.01}$$

$$\Rightarrow A \cong 99.100$$

$$\Rightarrow \boxed{A \cong 99\ km^2}$$

+1

9. (8 points) Evaluate the following definite integral exactly using the Fundamental Theorem of Calculus. Show your work. A calculator solution will earn no credit.

$$\int_0^{\pi} (2\sin(x) + 4)\,dx.$$

$$= \underbrace{-2\cos(x)}_{+2} + \underbrace{4x}_{+2} \Big|_0^{\pi}$$

$$= -2\cos(\pi) + 4\pi - (-2\cos(0) + 4(0))$$

$$= -2(-1) + 4\pi + 2$$

$$= 4 + 4\pi = \boxed{4(1+\pi)}$$

+4

10. (8 points) Over the past fifty years the carbon dioxide level in the atmosphere has increased. If $C(t)$ is the carbon dioxide level in parts per million (ppm) and t is time in years since 1980, the rate of change of the carbon dioxide level is modeled by

$$\frac{dC}{dt} = 1.24 + 0.03t.$$

Find $C(t)$ if the carbon dioxide level was 339 ppm in 1980.

$$C = \int (1.24 + 0.03t)\,dt \quad +2$$

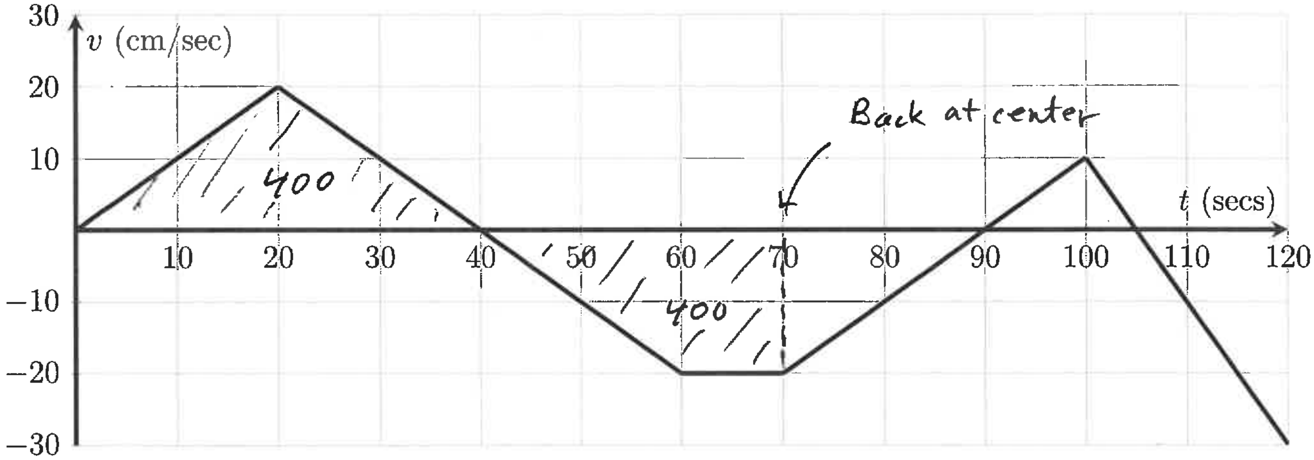
$$= 1.24t + 0.015t^2 + C \quad +2$$

$$339 = C(0) = C \quad +2$$

$$\therefore \boxed{C(t) = 1.24t + 0.015t^2 + 339}$$

+2

11. A mouse moves back and forth in a tunnel, attracted to bits of cheese alternatively introduced to and removed from the right and left ends of the tunnel. The mouse starts at the center of the tunnel at $t = 0$, and the graph of the mouse's velocity is given in the graph below, with positive velocity corresponding towards the right end of the tunnel.



(a) (1 point) How many times does the mouse change directions in the tunnel?

$\boxed{3 \text{ times}}$ +1

(b) (1 point) At what time is the mouse moving most rapidly to the right?

$\boxed{t = 20}$ +1

(c) (4 points) How many centimeters away from the center of the tunnel is the mouse at time $t = 100$ seconds, and it is to the right or left of the center at this time?

$\frac{1}{2}(40)(20) - \frac{1}{2}(20)(50+10) + \frac{1}{2}(10)(10)$ +2

$= 400 - 600 + 50$

$= -200 + 50 = -150$ +1

$\therefore$ The mouse is 150 cm to the left of the center +1

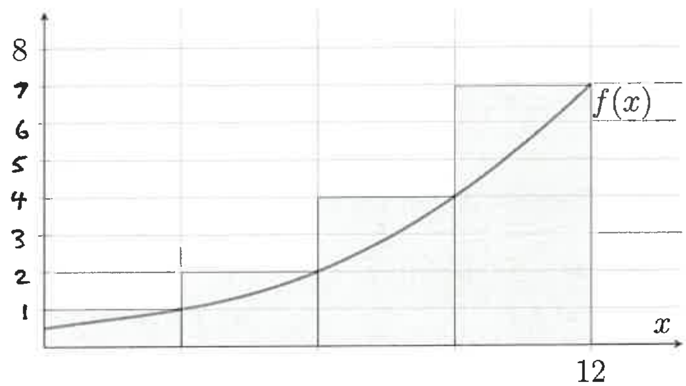
(d) (3 points) At which non-zero time t is the mouse at the center of the tunnel?

$\boxed{t = 70 \text{ sec.}}$ +1

$400 - \frac{1}{2}(20)(20) - 10(20)$

$= 0$ +2

12. The figure shows graph of a function $f(x)$ for $0 \leq x \leq 12$ along with rectangles used to approximate the definite integral $\int_0^{12} f(x)\,dx$.



(a) (2 points) Do the rectangles represent a left or a right Riemann sum?

$\boxed{\text{Right sum}}$ +2

(b) (2 points) What is the value of n, the number of subdivisions used?

$\boxed{n = 4}$ +2

(c) (2 points) What is the value of Δx?

$\boxed{\Delta x = 3}$ +2

(d) (3 points) Use the rectangles to estimate the value of $\int_0^{12} f(x)\,dx$.

$$R(4) = 3(\underbrace{1 + 2 + 4 + 7}_{+2})$$
$$= 3(14)$$
$$= \boxed{42}$$

+1

Formula Page

- $\frac{d}{dx}(cf(x)) = cf'(x)$
- $\frac{d}{dx}(f(x) \pm g(x)) = f'(x) \pm g'(x)$
- $\frac{d}{dx}(f(x) \cdot g(x)) = f'(x)g(x) + f(x)g'(x)$
- $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$
- $\frac{d}{dx}(c) = 0$, if c is a constant
- $\frac{d}{dx}(x^n) = nx^{n-1}$
- $\frac{d}{dx}(a^x) = a^x \cdot \ln(a)$
- $\frac{d}{dx}(e^x) = e^x$
- $\frac{d}{dx}(e^{kx}) = k \cdot e^x$, if k is a constant
- $\frac{d}{dx}(\sin(x)) = \cos(x)$
- $\frac{d}{dx}(\cos(x)) = -\sin(x)$

- $\int k\,dx = kx + C$, if k is a constant
- $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$, when $n \neq -1$
- $\int a^x\,dx = \frac{a^x}{\ln(a)} + C$
- $\int e^x\,dx = e^x + C$
- $\int \sin(x)\,dx = -\cos(x) + C$
- $\int \cos(x)\,dx = \sin(x) + C$
- $\int \frac{1}{x}\,dx = \ln(|x|) + C$
- $\int k\,dx = kx + C$, if k is a constant
- $\int c\,f(x)\,dx = c\int f(x)\,dx$, if c is a constant
- $\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$